MATH-951 Mathematical Modelling-II

Credit Hour: 3-0

Prerequisites: MATH-822 Mathematical Modelling-I

Course objectives: This course introduces powerful mathematical modeling techniques with reference to specific problems in physics, engineering, ecology, biology, sociology and economics, using dimensional analysis. It requires some background in differential equations, linear algebra and a little matrix theory. The purpose is that students taking this course should be able to construct models and use them to obtain results for the problems modeled.

Core Contents: The course will review linear algebra, matrix algebra and systems of differential equations. It will cover transformation of units and the structure of physical variables, dimensional analysis, dimensional similarities and models law; the nature of mathematical modeling; qualitative behavior of both linear and nonlinear system, stability analysis and bifurcation of dynamical systems; terminology and solution of several differential equations models, general equilibrium solutions of some realistic models; chaos in deterministic continuous systems.

Detailed Course Contents: Mathematical Preliminaries, Matrices and Determinants, Rank of a Matrix, System of linear equations, Review of dimensional systems, transformation of dimensions, arithmetic of dimensions, structure of physical variables, number of sets of dimensionless product of variables.

Sequence of Variables in Dimension Set: Dimensional physical variables is present, physical variable of identical dimensions are present, independent and dependent variables.

Dimensional Modeling: Introductory remarks, homology, specific similarities.

Dimensional Modeling: Dimensional similarities, models law, Categories and relations, scale effect. Linear Equation and Models: Some linear models, linear equations and their solution, homogenous and non-homogenous equations and their applications, dynamics of linear equation, some empirical models.

Nonlinear Equations and Models: Some nonlinear models, autonomous equations and their dynamics, Cobwebbing, derivatives and dynamics, some mathematical applications, bifurcation and period- doubling.

Modeling Change One Step at a Time: Introduction, compound interest and mortgage payments, some examples, compounding continuously.

Differential Equation Models: Carbon dating, age of the universe, HIV modeling.

Modeling in Physical Science: Introduction, calculus, Newton, and Leibniz, Rewriting Kepler's laws mathematically, generalization.

Learning Outcomes: Students are expected to understand:

Fundamentals of mathematical modeling.

Linear equation and models based on linear equations.

Non-linear equation and models based on non-linear equations.

Modeling change one step at a time.

Text Books:

F. R. Marotto (FRM), Introduction to Mathematical Modeling Thomson Brooks, 2006.

K.K. Tung (PET), Topics in Mathematical Modeling, Princeton University Press, 2007.

Thomas Szirtes (TS), Applied Dimensional Analysis and Modeling (Second Edition), Elsevier Inc., 2007.

Weekly Breakdown		
Week	***Chapt.***	***Topics***
1	T S 1	Mathematical Preliminaries, Matrices and Determinants, Rank of a Matrix, System of linear equations.
2	3,4, 5,7,10	Review of dimensional systems, transformation of dimensions, arithmetic of dimensions, structure of physical variables, number of sets of dimensionless product of variables.
3	14	Sequence of Variables in Dimension Set: Dimensional physical variables is present, physical variable of identical dimensions are present, independent and dependent variables
4-5	17	Dimensional Modeling: Introductory remarks, homology, specific similarities.
6	17	Dimensional Modeling: Dimensional similarities, models law, Categories and relations, scale effect.
7	FRM 2	Linear Equation and Models: Some linear models, linear equations and their solution, homogenous and non-homogenous equations and their applications, dynamics of linear equation, some empirical models.
8	3	Nonlinear Equations and Models: Some nonlinear models, autonomous equations and their dynamics, Cobwebbing, derivatives and dynamics, some mathematical applications, bifurcation and period-doubling.
9	**Mid Semester Exam**	
10	KKT 3, 4	Modeling Change One Step at a Time: Introduction, compound interest and mortgage payments, some examples, compounding continuously. Differential Equation Models: Carbon dating, age of the universe, HIV modeling.

11	5	Modeling in Physical Science: Introduction, calculus, Newton, and Leibniz, Rewriting Kepler's laws mathematically, generalization.
12	6	Nonlinear Population Models: An introduction to qualitative analysis using phase planes, population models, harvesting models, economic considerations, depensation growth models
13	7, 8	Discrete Time Logistic Map, Periodic and Chaotic Solutions: Logistic growth for non-overlapping generations, discrete map, sensitivity to initial conditions.Snowball Earth and Global Warming: Introduction, simple climate models, the equilibrium solutions
14	8	Snowball Earth and Global Warming: Stability, the global warming controversy, a simple equation for climate perturbation, solution of equilibrium global warming
15	10	Marriage and Divorce: Mathematical models of self-interaction and martial interaction, an example of validating couple, terminology, general equilibrium solutions.
16	11	Chaos in Deterministic Continuous System: Introduction, some history of Henri and Lorenz, the Lorenz equations as model of convection, chaotic waterwheel.
17		Review
18	**End Semester Exam**	